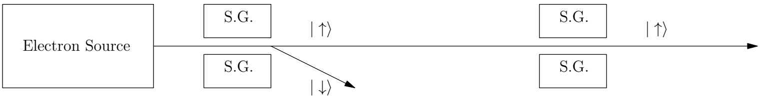
Transition Magnetic Moment and Collective Neutrino Oscillations

Shashank Shalgar

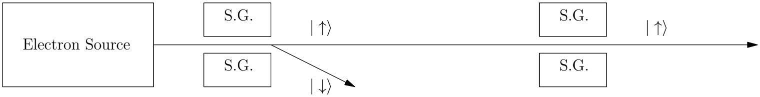
Northwestern University/University of New Mexico

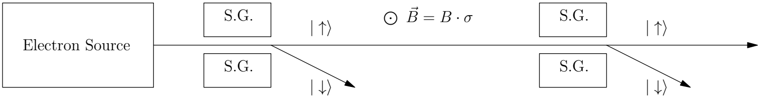
arXiv:1207.0516 and 1301.5637
(work done in collaboration with André de Gouvêa)

Introduction(Familiar system)



Introduction(Familiar system)





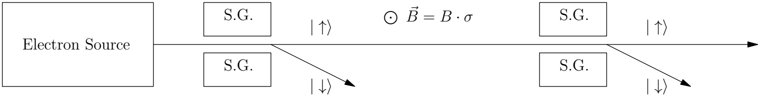
$$[S_Z, B \cdot \sigma] \neq 0$$

Equations of motion

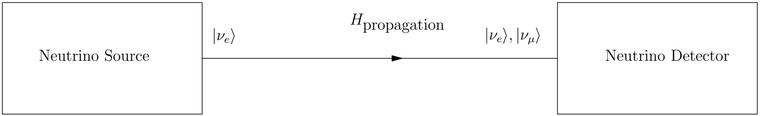
$$\rho = \begin{pmatrix} \langle \psi_\uparrow^* \psi_\uparrow \rangle & \langle \psi_\uparrow^* \psi_\downarrow \rangle \\ \langle \psi_\downarrow^* \psi_\uparrow \rangle & \langle \psi_\downarrow^* \psi_\downarrow \rangle \end{pmatrix}$$

$$\begin{aligned} \rho(t) &= e^{-iHt} \rho(0) e^{iHt} \\ &= e^{-iHt} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} e^{iHt} \\ \text{Remember}: \quad H &= B \cdot \sigma \end{aligned}$$

Neutrino Oscillations



$$P(|\uparrow\rangle \rightarrow |\downarrow\rangle) \neq 0 \Rightarrow [S_Z, B \cdot \sigma] \neq 0$$



$$P(|\nu_e\rangle \rightarrow |\nu_\mu\rangle) \neq 0 \Rightarrow [H_{\text{weak}}, H_{\text{propagation}}] \neq 0$$

Equations of motion

$$\rho = \begin{pmatrix} \langle \psi^*_{\nu_e} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_e} \psi_{\nu_\mu} \rangle \\ \langle \psi^*_{\nu_\mu} \psi_{\nu_e} \rangle & \langle \psi^*_{\nu_\mu} \psi_{\nu_\mu} \rangle \end{pmatrix} \quad \rho^c = \begin{pmatrix} \langle \psi^{c*}_{\nu_e} \psi^c_{\nu_e} \rangle & \langle \psi^{c*}_{\nu_e} \psi^c_{\nu_\mu} \rangle \\ \langle \psi^{c*}_{\nu_\mu} \psi^c_{\nu_e} \rangle & \langle \psi^{c*}_{\nu_\mu} \psi^c_{\nu_\mu} \rangle \end{pmatrix}$$

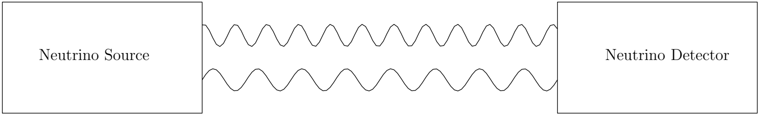
$$\rho(t) = e^{-iHt} \rho(0) e^{iHt}$$

$$H = \begin{pmatrix} -\omega \cos(2\theta) & \omega \sin(2\theta) \\ \omega \sin(2\theta) & \omega \cos(2\theta) \end{pmatrix} \quad \omega = \frac{m_2^2 - m_1^2}{4E}$$

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta) sin^2(\omega L)$$

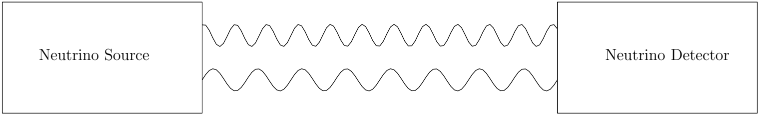
P independent of sign of ω

Physical picture

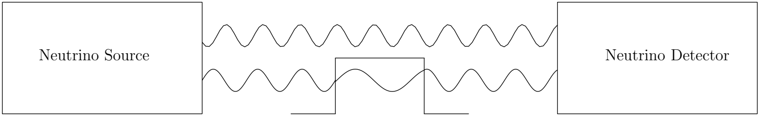


Can external factors affect the relative phase?

Physical picture



Can external factors affect the relative phase?



Matter effect is sensitive to sign of ω

Matter modified Hamiltonian

$$H = \begin{pmatrix} -\omega\cos(2\theta) & \omega\sin(2\theta) \\ \omega\sin(2\theta) & \omega\cos(2\theta) \end{pmatrix} + \begin{pmatrix} \pm\sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix}$$

Equations of motion for are dependent on the sign of ω

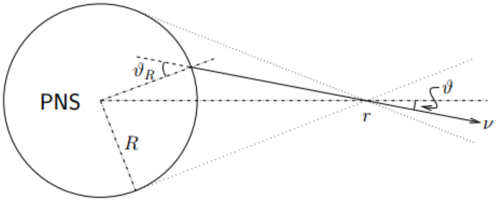
Self Interactions

$$H_{self} = \sqrt{2} G_F n_\nu \int dE \left(\rho(E) - \rho(E)^{c*}\right) + \mathrm{Tr}\left(\left(\rho(E) - \rho(E)^{c*}\right)\right)$$

If we know the initial flux (temperature and chemical potential) of the neutrinos we can calculate the flux that will be seen on the earth.

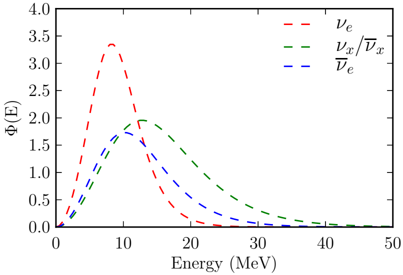
Including self-interactions makes the equation of motion very sensitive to the sign of ω.

Multi-angle effects



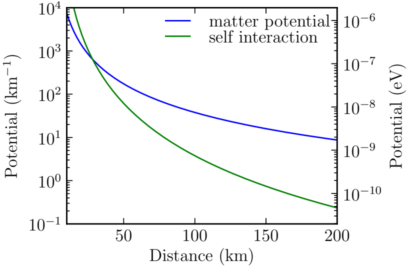
H_{self} is a function of θ.

Initial flux



M. T. .Keil, G. G. Raffelt and H. -T. Janka, Astrophys. J. **590**, 971 (2003) [astro-ph/0208035]

Matter and self-interaction potential



Final flux

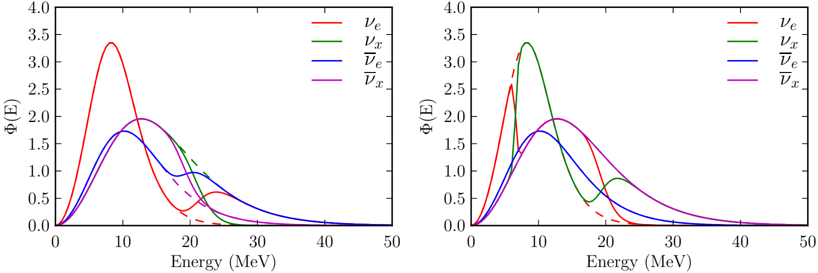
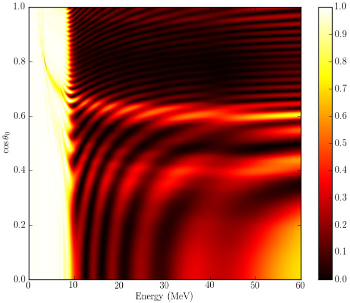
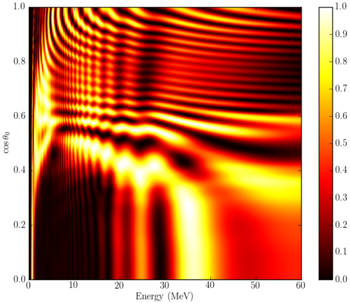


Figure: Initial and final fluxes for normal (left) and inverted hierarchy (right). The initial(final) flux spectra are denoted by dashed(solid) lines.

Multiangle plots

Survival probability of electron neutrino for inverted hierarchy

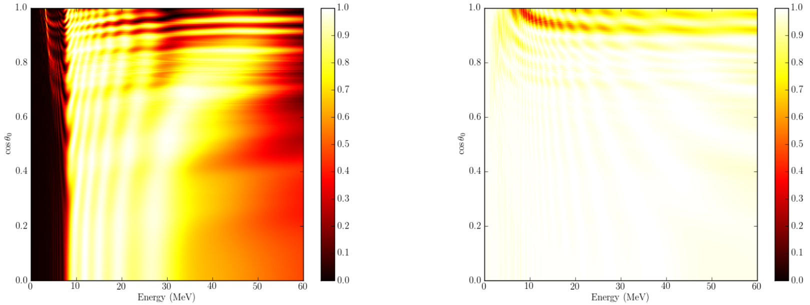




Reproduction of results from H. Duan, G. M. Fuller, J. Carlson and Y. -Z. Qian, “Coherent Development of Neutrino Flavor in the Supernova Environment,” Phys. Rev. Lett. **97**, 241101 (2006) [astro-ph/0608050]

Multiangle plots

Survival probability of electron neutrino for normal hierarchy



Reproduction of results from H. Duan, G. M. Fuller, J. Carlson and Y. -Z. Qian, "Coherent Development of Neutrino Flavor in the Supernova Environment," Phys. Rev. Lett. **97**, 241101 (2006) [astro-ph/0608050]

"Switch on" effect

- matter effect (electron) $\propto$ mixing angle θ
- self interactions: very sensitive to sign of ω for even a very small value of θ
- Is this the case only for θ_{13}?
- Does it matter whether neutrinos are Majorana or Dirac?

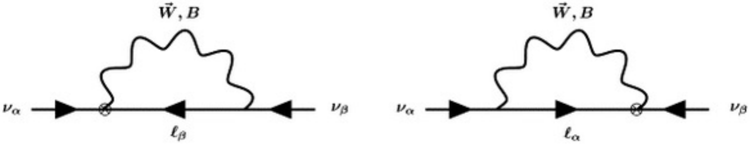
Magnetic moment

It determines the rate of $\nu_{iL} \to \nu_{jR}$ due to interaction with electromagnetic fields

$$\left.\begin{array}{c} \mu_{ij}^{\mathrm{D}} \\ \epsilon_{ij}^{\mathrm{D}} \end{array}\right\} = \frac{eG_F}{8\sqrt{2}\pi^2}\,(m_i \pm m_j) \sum_{l=e,\mu,\tau} f\left(\frac{m_l^2}{m_W^2}\right) U_{li}^* U_{lj}$$

$$\mu_{ii}^{\mathrm{D}} \simeq 3.2 \times 10^{-19} \left(\frac{m_i}{\mathrm{eV}}\right) \mu_B$$

Transition magnetic moment



S. Davidson, M. Gorbahn and A. Santamaria, Phys. Lett. B **626**, 151 (2005)

$$\left.\begin{matrix}\mu^{\mathrm{D}}_{ij} \\ \epsilon^{\mathrm{D}}_{ij}\end{matrix}\right\} \simeq -4 \times 10^{-23} \left(\frac{m_i \pm m_j}{\mathrm{eV}}\right) \sum_{l=e,\mu,\tau} \left(\frac{m_l}{m_\tau}\right)^2 U^*_{li} U_{lj} \; \mu_B$$

Majorana transition magnetic moment

If ν_i and ν_j have same $\mathcal{CP}$ phase

$$\mu_{ij}^M = 0 \qquad \epsilon_{ij}^M = 2\epsilon_{ij}^D$$

If ν_i and ν_j have same $\mathcal{CP}$ phases

$$\mu_{ij}^M = 2\mu_{ij}^D \qquad \epsilon_{ij}^M = 0$$

$$\begin{aligned} \text{Present Limits}: \quad \mu_{ii}^D &\lesssim 10^{-12}\mu_B \\ \mu_{ij}^M &\lesssim 10^{-10}\mu_B \end{aligned}$$

Particle Data Group, K. Nakamura et al., J.Phys.G G**37**, 075021 (2010)

Vacuum Hamiltonian

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} & \rho_{e\bar{e}} & \rho_{e\bar{x}} \\ \rho_{xe} & \rho_{xx} & \rho_{x\bar{e}} & \rho_{x\bar{x}} \\ \rho_{\bar{e}e} & \rho_{\bar{e}x} & \rho_{\bar{e}\bar{e}} & \rho_{\bar{e}\bar{x}} \\ \rho_{\bar{x}e} & \rho_{\bar{x}x} & \rho_{\bar{x}\bar{e}} & \rho_{\bar{x}\bar{x}} \end{pmatrix}$$

$$H_{vac} = \begin{pmatrix} -\omega\cos 2\theta & \omega\sin 2\theta & 0 & \mu B_T \\ \omega\sin 2\theta & \omega\cos 2\theta & -\mu B_T & 0 \\ 0 & -\mu B_T & -\omega\cos 2\theta & \omega\sin 2\theta \\ \mu B_T & 0 & \omega\sin 2\theta & \omega\cos 2\theta \end{pmatrix}$$

Self-interactions

$$H_{self} = \sqrt{2} G_F n_\nu \int dE \; G^\dagger(\rho(E) - \rho(E)^{c*}) G + \frac{1}{2} G^\dagger \mathrm{Tr}\left((\rho(E) - \rho(E)^{c*}) G\right)$$

where, $G = \mathrm{diag}(1, 1, -1, -1)$.

arXiv:1207.0516 and 1301.5637 (work done i

What is the magnetic field?

The core collapse causes the magnetic flux to be compressed in to a very small volume ($B \sim 10^{12}$ gauss)
We use the following magnetic field

$$B(r) = \left(\frac{50}{r(km)}\right)^2 10^{12} gauss$$

There is no way of telling the direction of the magnetic field. What happens if we assume it to be in the transverse direction?

Effect of transition magnetic moment($\theta = 0$)

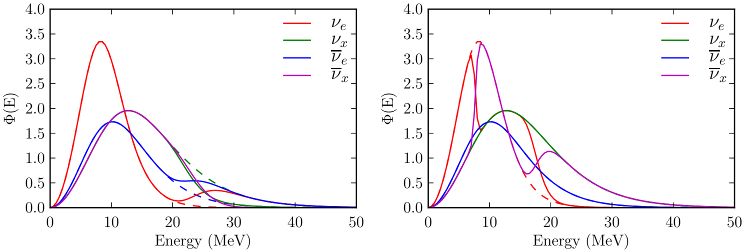


Figure: Initial and final flux spectra including the effect of transition magnetic moment for inverted(left) and normal(right) with hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-2}(\mu_{\nu_D} B)_{sm}$ and $\theta = 0$

Final flux (with magnetic moment)

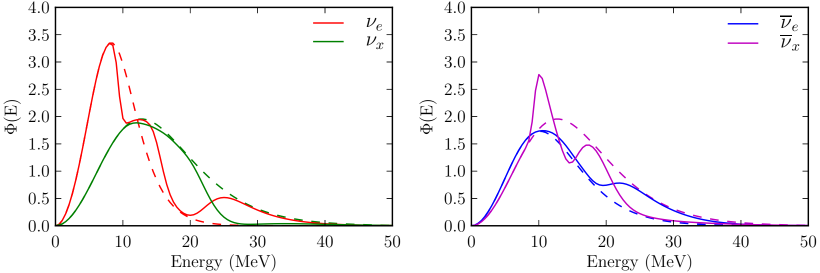


Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with normal hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-2}(\mu_{\nu_D} B)_{sm}$

Final flux (with magnetic moment)

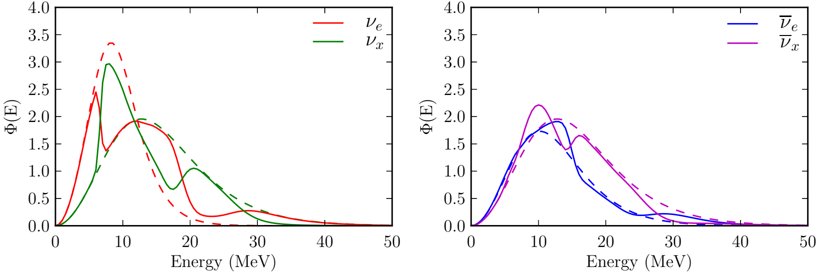


Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-2}(\mu_{\nu_D} B)_{sm}$

Conclusions and future direction

- Collective Oscillations: It is the only known phenomenon to our knowledge where transition magnetic moment of the order predicted by Standard Model could have phenomenological consequences
- We don't know whether it is practical to measure or interpret non-vanishing transition magnetic moments by observing supernova neutrino fluxes
- Will the multi-angle calculations lead to qualitatively different results?
- How will this effect influence r-process nucleosynthesis?

Multi-angle effects

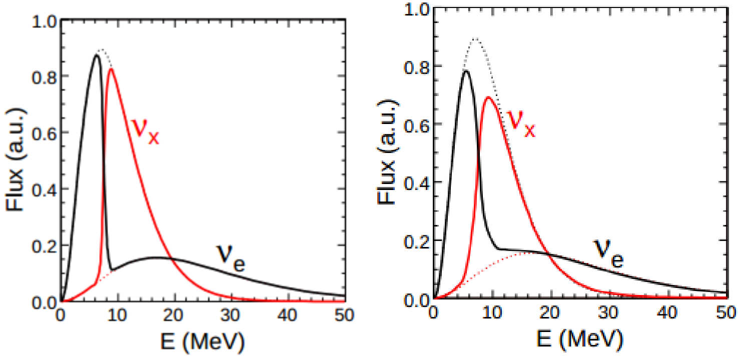
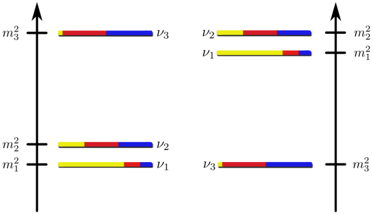


Figure: Final flux for single-angle(left) and multi-angle(right) calculation (G. L. Fogli, E. Lisi, A. Marrone and A. Mirizzi, JCAP **0712**, 010 (2007) [arXiv:0707.1998 [hep-ph]].

Three flavor Hamiltonian



$$H_{vac} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{12}^2 & 0 \\ 0 & 0 & \Delta m_{13}^2 \end{pmatrix} U^\dagger$$

Three flavor Hamiltonian

$$H_{vac} = \begin{pmatrix} H_\theta & H_{\mu B} \\ -H_{\mu B} & H_\theta \end{pmatrix}$$

where, H_θ and $H_{\mu B}$ are given below

$$\begin{aligned} H_\theta &= U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{12}^2 & 0 \\ 0 & 0 & \Delta m_{13}^2 \end{pmatrix} U^\dagger \\ H_{\mu B} &= \begin{pmatrix} 0 & \mu_{e\mu} B & \mu_{e\tau} B \\ -\mu_{e\mu} B & 0 & \mu_{\mu\tau} B \\ -\mu_{e\tau} B & -\mu_{\mu\tau} B & 0 \end{pmatrix} \end{aligned}$$

Three flavor calculations

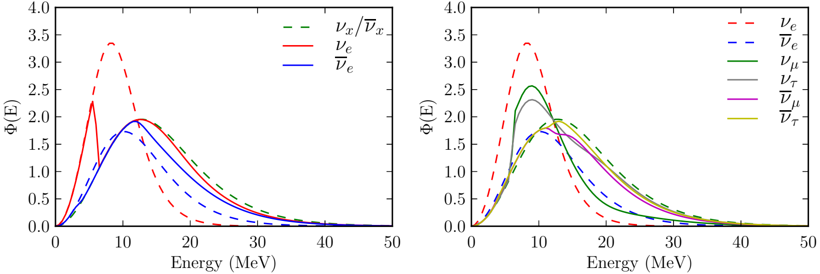


Figure: Initial and final flux spectra without including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 0$

arXiv:1207.0516 and 1301.5637 (work done i

Three flavor calculations

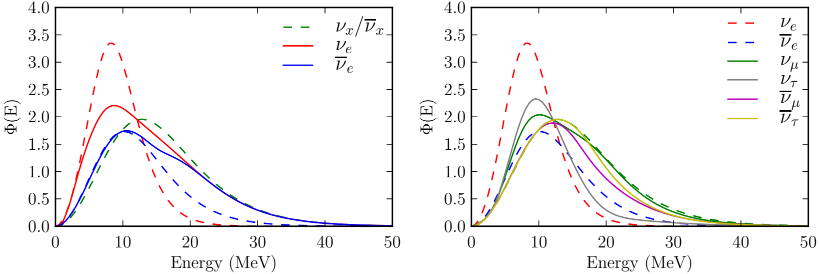
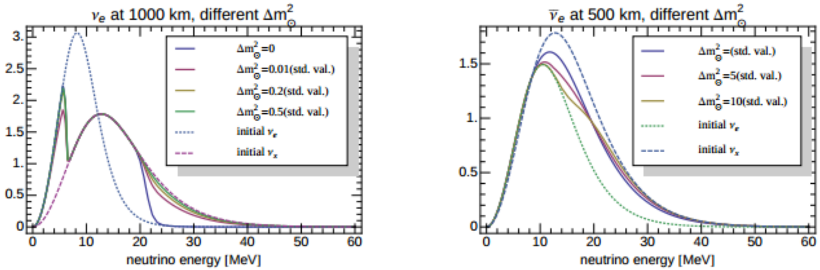


Figure: Initial and final flux spectra without including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with normal hierarchy. In this simulation we have used $\mu_\nu B(r) = 0$

why do we need three flavor calculations?



Two flavor approximation can lead to 'fake' instabilities

A. Friedland, Phys. Rev. Lett. **104**, 191102 (2010) [arXiv:1001.0996 [hep-ph]]

Three flavor calculations

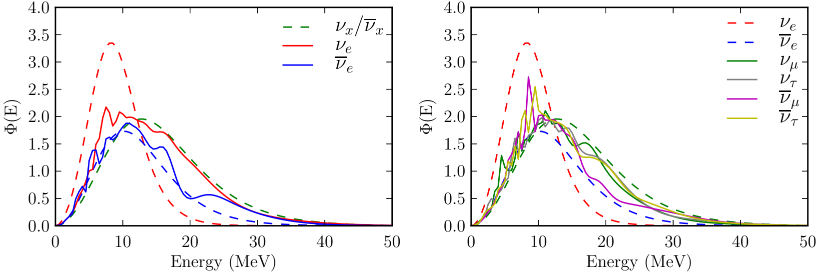


Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos(left) and anti-neutrino(right) with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$

Three flavor calculations

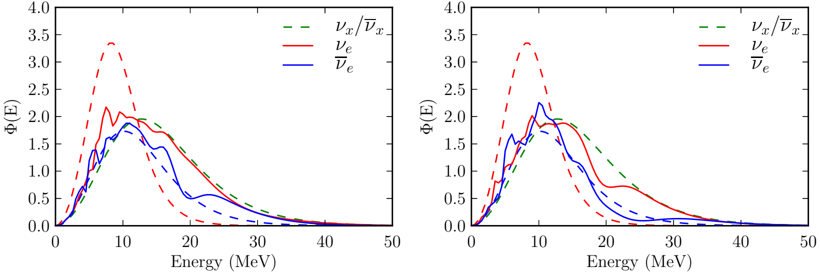


Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos with $\delta = 0^\circ$ and $\delta = 180^\circ$ with inverted hierarchy. In this simulation we have used $\mu_\nu B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$

Three flavor calculations

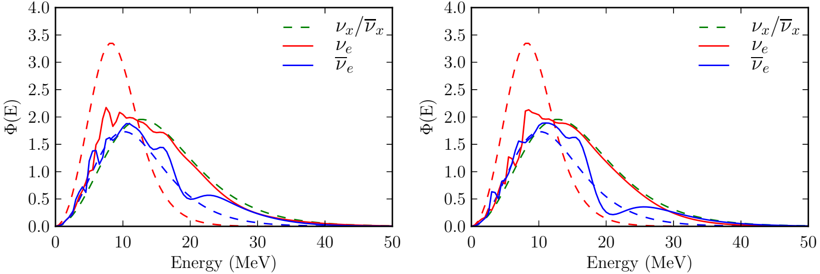


Figure: Initial and final flux spectra including the effect of transition magnetic moment for neutrinos with and with inverted hierarchy.
$\mu_{e\mu} B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$ (left) and $\mu_{\mu\tau} B(r) = 10^{-4}(\mu_{\nu_D} B)_{sm}$ (right)

arXiv:1207.0516 and 1301.5637 (work done i

MSW effect

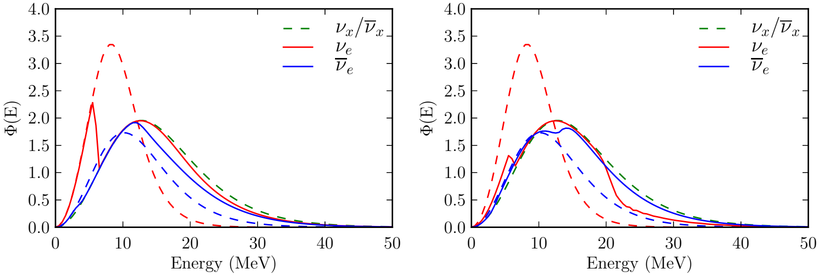


Figure: Flux before and after the standard MSW effect for inverted hierarchy.

MSW effect

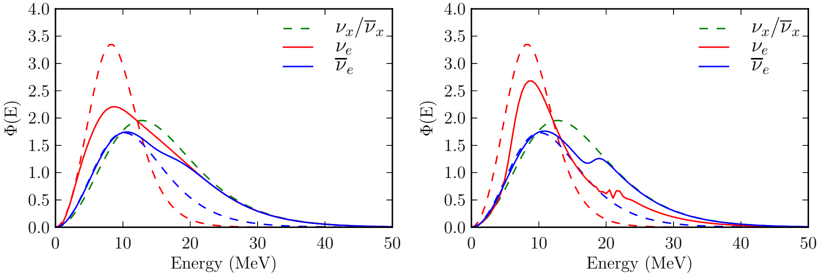


Figure: Flux before and after the standard MSW effect for normal hierarchy.

MSW effect

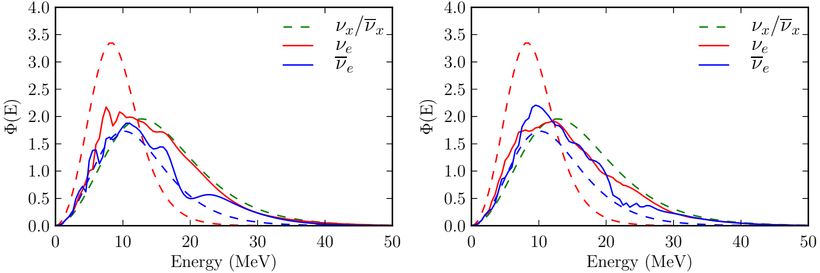


Figure: Flux before and after the standard MSW effect for inverted hierarchy. $\mu_{e\mu}B(r) = 10^{-4}(\mu_{\nu_D}B)_{sm}$

MSW effect

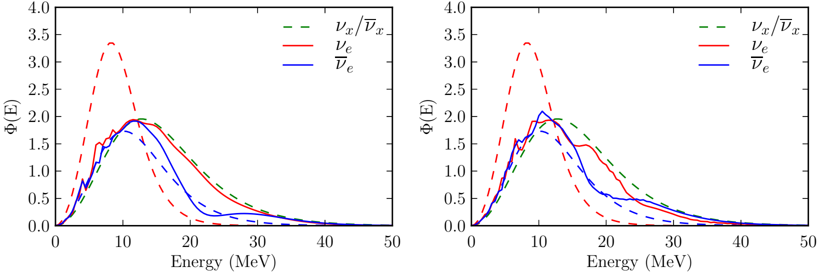


Figure: Flux before and after the standard MSW effect for normal hierarchy.
$\mu_{e\mu} B(r) = 10^{-4} (\mu_{\nu_D} B)_{sm}$

Conclusions

- The neutrino flux spectra for Majorana neutrinos is significantly different than Dirac neutrinos
- Transition magnetic moment of neutrinos can have a "switch-on" effect on $\nu_L \to \nu_R$ oscillations, just like θ_{13} for $\nu_e \to \nu_x$
- Neutrinos from galactic supernova(e) is the only phenomenon for which transition magnetic moments of the order predicted by standard model can have significant impact. This is the only known way of detecting non-zero transition magnetic moments.
- Multi-angle calculations with transition magnetic moment needed solid conclusions can be drawn.